Online Identification of Primary Social Groups

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Abstract. Online group identification is a challenging task, due to the inherent dynamic nature of groups. In this paper, a novel framework is proposed that combines the individual trajectories produced by a tracker along with a prediction of their evolution, in order to identify existing groups. In addition to the widely known criteria used in the literature for group identification, we present a novel one, which exploits the motion pattern of the trajectories. The proposed framework utilizes the past, present and predicted states of groups within a scene, to provide robust online group identification. Experiments were conducted to provide evidence of the effectiveness of the proposed method with promising results.

Keywords: social groups, group identification, online, motion prediction.

1 Introduction

Surveillance video applications have attracted the interest of the research community throughout the years. The majority of the related literature focuses on single object activity. Given the significant improvement of such methodologies and the need for higher level semantic extraction, the interest of the research community is shifting towards more complex structures i.e. groups. A group is defined as a collection of people who interact with one another, share similar characteristics and collectively have a sense of unity. Groups in which individuals intimately interact and cooperate over a long period of time are also known as "Primary Social Groups" [1]. Additionally to the vagueness of the "Group" definition, the high variation of the recording conditions renders group identification a challenging task.

In the proposed work, a novel online methodology to identify groups is presented. The definition of a primary group requires that its members interact for a significant amount of time, requiring a critical amount of evidence to be accumulated. In an online scenario, this evidence accumulation introduces a delay before declaring a group. To alleviate this delay, the motion of the group members in subsequent frames is predicted, using a motion model that is created offline, using accumulated motion priors. Exploiting, collectively, the already identified trajectories and the prediction of the trajectory evolvement, robust, online group identification is made possible. Acknowledging the errors introduced by trackers and challenges introduced by the scene characteristics, a probabilistic approach to identify a group is followed. Rather than taking hard decisions, the confidence that each individual belongs to a group is evaluated.

The main contribution of the proposed framework is that it exploits the predicted positions of individuals in subsequent frames, in addition to their present and past ones, to enable online group identification. The predictions made are based on prior accumulated trajectories from the scene, exploiting context awareness. A novel metric is introduced to assess trajectory similarity, which takes advantage of the motion pattern of the trajectories. It is observed that people forming a group follow a similar motion pattern through time. This pattern is captured, using the convex hull of the trajectories' points, and a similarity criterion is created, based on the area of convex hull.

The rest of the paper is organized as follows: in Section 2 related work on group analysis is presented. Section 3 describes the proposed methodology and in Section 4 the experimental results are drawn. Section 5 contains conclusions and discussion on the proposed method.

2 Related Work

Two main approaches have been proposed so far in the literature concerning group identification. The first approach considers groups as genuine atomic entities, without contemplating individual tracks, in an attempt to overcome people detection problems in highly cluttered scenes. The second approach detects and tracks individuals, and builds upon these findings towards group tracking. We briefly review literature based on these two main approaches.

Following the first approach, a tracking algorithm is developed in [2] that uses Correlated Topic Modeling (CTM) to capture different crowd behaviors in a scene. In [3], multiple-frame feature point detection and tracking is proposed, and crowd events are modeled for specific scenarios. In [4], Reisman et al. propose to use slices in the spatio-temporal domain to detect inward motion. Their system calculates a probability distribution function (PDF) for left and right inward motion and infers a decision for crowd detection, by thresholding left and right motion histograms. In [5], the authors create a crowd model using accumulated motion and foreground information. Occurrence PDF and orientation PDF are employed to find the most frequent path of the crowd.

Employing the latter approach, an agent-based behavioral model of pedestrians is proposed in [6]. An energy function is defined and its minimization leads to the estimation of pedestrian destination and social relationships (groups). In [7], the grouping between pedestrians is treated as a latent variable, which is estimated jointly together with the trajectory information. In [8], small groups of individuals travelling together are discovered by a bottom up hierarchical clustering, using a generalized, symmetric Hausdorff distance, defined with respect to pair-wise proximity and velocity. In [9], mobile objects in a scene are stored as moving region structures and the real groups are tracked by computing the moving regions' trajectories. An interpretation module recognizes the behavior of the tracked groups. In [10], a probabilistic grouping strategy is used. A path-based grouping scheme determines a soft segmentation of groups. Probabilistic models are derived to analyze individual track motion as well as group interactions.

3 Proposed Methodology

Due to the dynamic nature of groups, there are inherent difficulties in deciding the existence or evolution of a group, judging only from a single frame. In order to tackle the group identification problem, prior trajectories and sophisticated predictions of the trajectory evolvement are employed.

The proposed approach is examining a number of criteria related to the positions and trajectories of people present in the scene, to create a number of hypotheses regarding the possible existence of pairs, which are regarded as the building block of a group. A voting scheme is employed to decide upon the validity of the formed hypotheses. The future positions of the individuals are calculated using a prediction methodology [13], based on prior motion patterns in the specific scene. Previously validated relationships between people are propagated to the next frames and are tested again, using the future predicted positions of the respective people. Pairs with mutual individuals are merged to create larger groups. A general overview of the proposed framework is depicted in Figure 1.

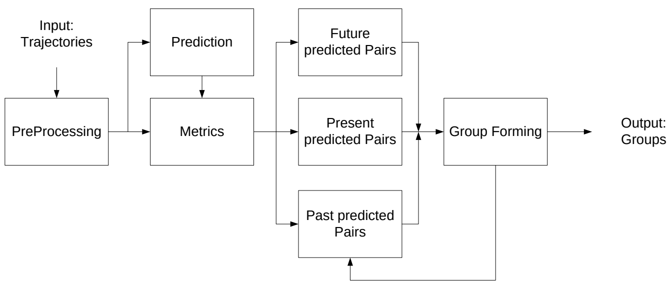

Fig. 1: System Overview.

3.1 Group Identification Criteria

People appearing in the scene are detected in every frame and are tracked throughout time. The trajectory T of an individual i is defined as a set of locations w.r.t. time:

$$\mathbf{T}_i = \{x_t, y_t\}_{t=1}^{K}, \tag{1}$$

where x_t and y_t are the (x, y) coordinates, respectively, of person i at frame t, and K is the length of the trajectory in frames. The extracted trajectories are filtered with a median filter in order to remove noise and produce smoother paths.

Fig. 2: Example of trajectories of individuals belonging in the same group. The convex hull of each trajectory is drawn around it.

Similar to [7, 8, 10], it is assumed that the trajectories of group members share some common characteristics, namely spatiotemporal proximity and velocity similarity. An additional characteristic, overlooked until now, is the fact that the trajectories of the group members have a similar motion pattern. The properties of the motion pattern are captured, defining the convex hull of the trajectory as its shape descriptor. An example can be seen in Figure 2. All the above criteria are used in the proposed framework to assess the similarity of the generated trajectories.

The spatiotemporal proximity between group members is the most common similarity metric used. In the proposed work, the notion of the proximity area is utilized. For every individual, an area is set, within which, every other individual is regarded as being close. Due to the perspective effect introduced by the camera setup, the size of this area depends on the distance between the individual and the camera. The area size is defined relative to the width and height of its bounding box (w_{bbox}, h_{bbox}), in a naive attempt to rectify the effect of the distance to the camera. Thus, the proximity area of person i can be defined as $proxArea_i = \{x_i, y_i, th_w, th_h\}$, where (x_i, y_i) is the center of the bounding box of person i, and (th_w, th_h) are the width and height of the proximity area. Two individuals are considered a group when they are in each other's proximity area for more than a percentage λ of their trajectory length:

$$\{(x_j, y_j) \in proxArea_i\}_{j=1}^{K_j} > \lambda K_j, \tag{2}$$

where $\{(x_j, y_j) \in proxArea_i\}$ is the number of the trajectory points of person j in the proximity area of person i, and K_j is the length of the trajectory of j. The value of λ is experimentally set to 0.7, in all cases, to ensure that the candidate pair remains close most of the time, allowing, though, robustness to small diversions and/or tracking failures.

To identify successfully a group, though, the relative position of the individuals is equally important to their relative distance. Consider a case where two persons are moving close enough, but one is in front of the other. These two

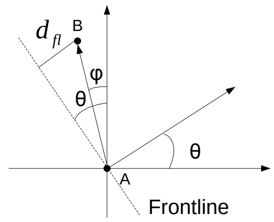

Fig. 3: Calculation of motion frontline.

individuals do not form a pair, even though they are close. In order to filter such cases, the notion of the motion frontline is introduced. An axis, perpendicular to the motion orientation of a person, is defined as its frontline, depicted in Figure 3. The distance of a second person from this axis (d_{fl}) is defined in (3).

$$d_{fl} = d_{p_A,p_B} \sin|(\theta + sign(\tan\phi)\cdot\phi)|, \tag{3}$$

where p_A, p_B are the positions of the two persons, d_{p_A,p_B} is their distance, θ is the motion orientation translated to $\exists[0, \pi/2]$ set, and ϕ is the angle formed by the vector **AB** and the Y axis, as depicted in Figure 3.

A threshold (th_{fl}) concerning the acceptable distance of a person to the others frontline, relative to the size of the bounding box of the first one, is confronted as an additional criterion for pair validation.

$$d_{fl} < th_{fl}\cdot(h_{bbox}\sin\theta + w_{bbox}\cos\theta), \tag{4}$$

Another criterion to assess the similarity of two trajectories is their velocity ($\mathbf{V}$) characteristics. Speed is an important similarity metric between trajectories. It is observed that individuals belonging to the same group have also similar speeds. In order to exploit this observation, we calculate the mean speed of every individual. Two individuals are assumed to have similar speeds if their speed ratio is below a threshold (th_{sp}):

$$\overline{v_x} = \frac{\sum_{l=2}^{K_i} x_l - x_{l-1}}{K_i - 1},\ \overline{v_y} = \frac{\sum_{l=2}^{K_i} y_l - y_{l-1}}{K_i - 1} \tag{5}$$

$$\frac{max(norm(\overline{v_x}_i, \overline{v_y}_i), norm(\overline{v_x}_j, \overline{v_y}_j))}{min(norm(\overline{v_x}_i, \overline{v_y}_i), norm(\overline{v_x}_j, \overline{v_y}_j))} < th_{sp}, \tag{6}$$

where $\overline{v_x}$ and $\overline{v_y}$ are the mean speed on X and Y axis, respectively, x_l and y_l are the coordinates on X and Y axis at the l-th frame of the trajectory, respectively, and K_i the length of trajectory. Moreover, $norm(\overline{v_x}_i, \overline{v_y}_i)$ and $norm(\overline{v_x}_j, \overline{v_y}_j)$ are the norm of the mean speed vector of person i and j, respectively.

The similarity in the orientation of the velocity is also a crucial criterion in group identification. It is defined as the angle formed between every point of the trajectory and its successive points, as depicted in Figure 4. In order to have an overall view of the orientation, the angle formed with every subsequent point of the trajectory is calculated and the mean value $\bar{a}$ is extracted. Since the arithmetic mean is not suitable for circular quantities [12], $\bar{a}$ is calculated using (8).

$$\bar{b_p} = arctan(\frac{1}{K-p} \cdot \sum_{q=p+1}^{K} \sin b_{q,p}, \frac{1}{K-p} \cdot \sum_{q=p+1}^{K} \cos b_{q,p}), \tag{7}$$

$$\bar{a} = arctan(\frac{1}{K-1} \cdot \sum_{l=1}^{K-1} \sin \bar{b_p}, \frac{1}{K-1} \cdot \sum_{l=1}^{K-1} \cos \bar{b_p}), \tag{8}$$

where $\bar{b_p}$ is the mean angle between point p and the subsequent trajectory points, K is the length of the trajectory, $b_{q,p}$ is the angle between point p and point q, and $\bar{a}$ is the mean angle. Two individuals are considered to have the same motion orientation if the difference between their mean angles is below a threshold ($th_{\bar{a}}$). The calculated mean angle differences $\exists[-\pi, \pi]$.

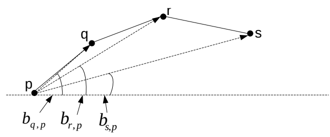

Fig. 4: Successive trajectory points and the formed angles.

Another important characteristic of the trajectories that has not been properly addressed in literature is the motion pattern. A novel similarity criterion based on it, is introduced in this framework; the shape similarity of the trajectory. The shape is defined as the convex hull of the trajectory points (see Figure 2). It is argued that the convex hull contains valuable information about the evolvement of the trajectory, capturing properties of the motion pattern followed. As we can see in Figure 2, the trajectories of the two people walking together have almost the same pattern and this is depicted also on their convex hulls. The area of the convex hulls is utilized to measure the similarity between them. Two individuals are considered to have similar motion patterns if the ratio of their convex hull areas is within certain limits (th_{ch}), as described in (9).

$$\frac{max(CH_i, CH_j)}{min(CH_i, CH_j)} < th_{ch}, \tag{9}$$

where CH_i and CH_j are the areas of the convex hulls of person i and j, respectively.

The final decision on the validity of a candidate pair is taken using a voting scheme. All the criteria described are tested, and an elementary decision is taken for each criterion $c_m \exists \{0, 1\}$, where m is the identifier of the metric, based on the thresholds defined. The final decision is the average value of all elementary votes. The voting function is described in the following formula:

$$confidence = \begin{cases} 0 & \text{if } c_d = 0 \| c_{\bar{a}} = 0 \\ \frac{1}{C} \sum_{m=1}^{C} c_m & \text{else} \end{cases}, \qquad (10)$$

where c_d is the vote of the distance criterion, $c_{\bar{a}}$ the vote of the orientation criterion, c_m the vote of criterion with label m, and C the number of criteria. The orientation similarity and the proximity between candidates are essential criteria of the group formation, and pairs that do not meet them are immediately excluded. For static people, only the proximity criterion is employed.

3.2 Prediction of Trajectory Evolution

In order to enable the online detection of groups, a prediction of the trajectory evolution is required to assist the validation of the group hypotheses created. The motion prediction methodology introduced in [13] is followed to provide the future positions of each individual in the current frame. It includes the offline, one-time creation of the motion models, and an online prediction module.

The first step in motion prediction is to create the motion model for the examined scene, based on prior motion patterns. The accumulated prior trajectories that are used as training material are divided into smaller tracklets with a fixed length $N_{tracklet}$. In order to use only the most informative tracklets, very small tracklets are removed and the remaining are filtered to produce smoother paths, producing a large set of tracklets that summarize the motion patterns observed in the scene. To reduce it, a grid of equally-spaced points is applied on the image. Mean shift clustering is performed on the local neighborhood of every grid point. The mean tracklet of every detected cluster is assigned on that point, and represents a local motion model. Thus, every grid point obtains multiple local motion models that reflect the underlying scene dynamics. Next, Gaussian Process (GP) [14] regression is used in order to model the dominant motion patterns.

The local motion models identified are exploited to create an online motion prediction module, extending the work of [13]. Given a person in the scene, a tracklet containing the $N_{tracklet}$ prior locations of the target is fed to the Motion Prediction module. This tracklet is assigned to a grid point of the scene, based on its localization. The motion models that correspond to this grid point are employed to estimate the next positions of the trajectory under investigation. Each model produces a predicted path that includes both the prior $N_{tracklet}$ locations and a set of subsequent predicted positions. This set of paths is filtered by removing non-fitting ones, choosing the most probable one.

3.3 Online Group Identification

A group is a dynamic structure that cannot be defined in a single frame. To identify groups, in an online fashion, an overlapping time window of N frames is defined, where the N_{th} frame is the current one. The remaining $N-1$ frames of the time window are the past frames. All frames after the N_{th} frame are considered as future ones.

The process of identifying groups is initiated from the current frame. The trajectories of all individuals in the defined time window are gathered, and their similarity is tested following the criteria described in Section 3.1. At the end of the voting procedure, a set of candidate pairs is identified. Then, for all individuals that have not been assigned a pair in the current frame, their pairing history in the last $N-1$ frames is examined. If they have constituted a pair with another individual for a significant amount of time and with high confidence, this pair is propagated to the current frame. Otherwise, pairs with short history and marginal confidence are discarded.

In order to boost the robustness of the proposed methodology, the pair hypotheses propagated, are tested using the evolution of the current trajectories, estimated using the motion prediction module described in Section 3.2. The similarity testing procedure described in Section 3.1 is applied for the predicted part of the trajectory and another set of candidate pairs is produced.

Finally, all the candidate pairs from the previous steps are combined to produce a final set. For every pair hypothesis based on history, it is examined whether it is propagated to the set of candidates based on prediction. If it does propagate, the pair's confidence value is updated to the mean of the history and prediction-based confidence. Otherwise, the pair's confidence is reduced to half of its initial value. This new set of candidate pairs is concatenated with the current frame's candidate pairs, and the final set of pairs is formed.

However, our goal is to identify groups and not just pairs of individuals. Therefore, all pairs with common individuals are merged so as to generate larger groups. The confidence value of the groups is calculated as the mean confidence of all included pairs.

4 Experimental Results

Our framework is implemented using MATLAB and tested on two datasets. All the similarity criteria thresholds employed, are presented in Table 1. Their value is set using a statistical analysis of the group characteristics in the same sequences used for training the motion models, and they are common for all datasets. The performance of the proposed group identification algorithm is evaluated, using Precision, Recall, and F-measure as metrics.

Metric	**Threshold values**
Proximity	$th_w = 4 \cdot w_{bbox}, \quad th_h = 0.8 \cdot h_{bbox}, \quad th_{fl} = 0.2$
Orientation	$th_{\bar{a}} = 30°$
Speed	$th_{sp} = 1.5$
Shape	$th_{ch} = 1.5$

Table 1: Thresholds employed in the validation criteria for pair hypotheses.

4.1 Dataset

For the evaluation process, two publicly available datasets are used, namely the BEHAVE dataset [16] and the dataset from the European Community (EC) funded project CAVIAR [15]. These two datasets were chosen because they have ground truth annotation regarding the trajectories of the people present in the scene. Additionally, for the BEHAVE dataset there is also annotation regarding the groups. For CAVIAR, the respective annotation was done manually by the authors, and it will be made publicly available to enable comparisons with other methodologies.

The BEHAVE dataset is outdoors, and it comprises of various people interaction scenarios. The frame rate is 25 frames per second (fps), and the resolution is $640x480$. A ground truth file of the annotated groups is also included. For our experiments Sequence 2 is used, which consists of 5700 frames.

The CAVIAR dataset is indoors (see Figure 1). It includes 26 video sequences, containing a varying number of individuals and groups. The average length of the video sequences is 1500 frames. The resolution of the frames is $384x288$ pixels and the frame rate of each sequence is at 25 fps.

For the experiments, the publicly available ground truth trajectories of both video datasets are used, to prevent the tracking errors from affecting the results, rendering future comparison even more difficult. Since most trackers do not track individuals whose bounding box has a width less than 24 pixels, these individuals were excluded from the ground truth.

4.2 Group Identification Results

We evaluate the group identification algorithm output of our framework using the group-related ground truth. The accuracy of our results has been evaluated, using as metrics Precision, Recall and F-measure at multiple levels, namely group, frame and total.

$$P_{Gf} = \frac{|\{relevantGroupMembers\} \cap \{retrievedGroupMembers\}|}{\{retrievedGroupMembers\}} \tag{11}$$

$$R_{Gf} = \frac{|\{relevantGroupMembers\} \cap \{retrievedGroupMembers\}|}{\{relevantGroupMembers\}} \tag{12}$$

$$P_{frm} = \frac{\sum_{f=1}^{N_G} P_{G_f}}{N_G}, \; R_{frm} = \frac{\sum_{f=1}^{N_G} R_{G_f}}{N_G}, \tag{13}$$

$$P = \frac{\sum_{l=1}^{N_{frm}} P_{frm}}{N_{frm}}, \; R = \frac{\sum_{l=1}^{N_{frm}} R_{frm}}{N_{frm}}, \tag{14}$$

$$F = 2\frac{PR}{P+R}, \tag{15}$$

where P_{G_f}, R_{G_f} are the precision and recall of the group f (G_f), N_G the total number of groups in frame frm, P_{frm}, R_{frm} are the overall precision and recall for frame frm, P and R are the overall precision and recall for the video sequence, respectively, N_{frm} is the number of frames of the video sequence that have at least two detected people, and F the F-measure, which combines precision and recall.

The video sequence of BEHAVE dataset contains the forming and deforming of groups of people. The video sequences of CAVIAR dataset contain different kind of group scenarios. From the 26 sequences 14 are chosen, since the rest do not contain groups.

The results of the video sequences, using the adopted metrics are presented in Table 2. Examples of correct group identification for the BEHAVE and CAVIAR dataset are depicted in Figures 6 and 7, respectively. The yellow bounding box in Figure 6c implies the change of the status of the group, since a new member has been added. As it can be seen, our algorithm produces accurate results in most cases. Group identification failures are usually due to sudden re-positioning of the group members within its limits. An example of such case is depicted in Figure 5, where a member of a group suddenly changes its intra group position (Figure 5b). Failures of this type are temporal and when the new ordering is finalized, the group is again correctly identified (Figure 5c).

(a) (b) (c)

Fig. 5: A group of 3 people (a) is correctly identified, (b) until one changes position and the grouping fails to include him, (c) and he is again included when he establishes a new position in the group.

(a) (b) (c)

Fig. 6: Example of group identification from the BEHAVE dataset

Comparison with other methodologies was not made possible, due to the different datasets which are not always available. Moreover, ground truth annotation is not provided and no standard evaluation metrics are employed. In

Dataset	Sequence	GT Group	Precision	Recall	F-measure
CAVIAR	**c2es1**	3	0.9757	0.9817	0.9788
	c2es2	3	0.9444	0.8342	0.8859
	c2es3	3	0.8953	0.8940	0.8946
	c2ls1	1	0.9957	0.9957	0.9957
	c2ls2	1	0.9815	0.9815	0.9815
	c3ps1	2	0.9923	0.9918	0.9920
	c3ps2	3	0.9501	0.8534	0.8992
	ceecp1	1	0.99	0.99	0.99
	cosow1	2	0.8880	0.8880	0.8880
	cosow2	5	0.9161	0.7748	0.8395
	csa1	1	0.8696	0.971	0.9175
	csa2	3	0.9459	0.9377	0.9418
	cwbs1	1	0.99	0.99	0.99
	cosme1	5	0.8691	0.9463	0.9060
BEHAVE	**Seq. 2**	11	0.9643	0.9310	0.9474

Table 2: Precision, Recall, and F-measure results for the sequences tested.

(a) (b) (c)

Fig. 7: Example of group identification from the CAVIAR dataset

this work, publicly available datasets and ground truth annotation (including the one that the authors will make public) are used, to encourage the research community to produce comparable results.

5 Conclusions and Discussion

A novel approach for online primary social group identification is presented. The framework proposed combines tracking information for each individual in the present and recent past with a prediction of their trajectory in the near future, for robust group identification. The prediction is based on a model trained with trajectories that have been accumulated from the examined scene and used as training set. For the identification of the groups, a novel criterion based on the motion pattern is combined with established ones. The effectiveness of the proposed framework is demonstrated on two publicly available datasets. Ground truth annotation for groups will be made available by the authors. Further validation is necessary to examine the effectiveness of our framework in more complicated group scenarios and camera settings.

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