Using Ellipsoidal Harmonics for 3D Shape Representation and Retrieval

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Abstract

A novel approach for 3D Shape description, applicable for search and retrieval applications, based on the theory of Ellipsoidal Harmonics is presented in this paper. The Ellipsoidal Harmonics are appropriately adopted in order to describe volumetric represented 3D objects. The experimental results, performed in a complete 3D object database, prove the efficiency of the proposed approach.

Keywords: 3D shape description, 3D object retrieval, Ellipsoidal Harmonics

Introduction

3D object retrieval is a relatively new and very challenging research field and a major effort of the research community has been devoted to the formulation of accurate and efficient 3D object search and retrieval algorithms. Generally, descriptor extraction procedures aim to describe the geometry of the 3D object. The commonly utilized descriptors are global features, local features, histograms, topological features or combinations of the above. Some very good surveys on this topic can be found in [1, 2].

The existing methods can be classified into three broad categories: the topology-based approaches, where the object is mainly described using topological features [3], the geometry-based approaches, where either global or local geometric descriptors are computed from the 3D object's geometry, and the view-based approaches. Global-based approaches present significantly higher discriminative power and are classified among the best 3D content-based retrieval approaches.

A major problem in the field of the 3D object retrieval is the invariance of 3D object description under the presence of rotation. Two methods have been proposed so far in the literature: the native rotation invariant descriptors (e.g. Spherical Harmonics [4]) and the rotation normalization. Both approaches present benefits and drawbacks. Although rotation normalization approach results in descriptors with higher discriminative power, the normalization is not robust (similar objects are not normalized in a similar manner [5]). In contrast, the native rotation invariant approaches, usually involve integrations, which results in lower discriminative power.

In this paper, a new geometric descriptor is proposed, which is based on the Ellipsoidal Harmonics. Ellipsoidal Harmonics offer a compact and discriminative representation of volume based 3D objects that is appropriate for 3D content-based search and retrieval.

Ellispoidal Harmonic Descriptor

The ellipsoidal harmonics are special functions that have been utilized in the field of astronomy [6] for describing surrounding force-fields of non-spherical objects. In this paper, the theory of Ellipsoidal Harmonics is adapted and applied to the field of 3D shape analysis and description. The basic motivation behind the selection of Ellipsoidal Harmonics for 3D object description relies on the intuition that an ellipsoid forms a better approximation of the shape of a 3D object when compared to the approximation using spheres. Detailed mathematical information concerning Ellipsoidal Harmonics can be found in [7].

The Ellipsoidal Harmonics are computed in Ellipsoidal Coordinate system. The ellipsoidal coordinates of a point (x, y, z) are given by the solution of

$$\frac{x^2}{\lambda_i^2} + \frac{y^2}{\lambda_i^2 - h^2} + \frac{z^2}{\lambda_i^2 - k^2} = 1 \tag{1}$$

with respect to λ_i^2. This results in 3 discrete solutions, $\lambda_1^2 \in [k^2, +\infty)$, $\lambda_2^2 \in [h^2, k^2]$ and $\lambda_3^2 \in [0, h^2]$. For a given (h, k), the family of ellipsoids, obtained for different values of λ_1^2 , are homofocal. The equations $\lambda_1^2 = const$, $\lambda_2^2 = const$ and $\lambda_3^2 = const$ define an ellipsoid, a hyperboloid of one sheet and a hyperboloid of two sheets respectively.

Ellipsoidal Harmonics $E_n^p(.)$ are the solutions of the Laplace equation in ellipsoidal coordinates $\nabla^2 V = 0$. There are four families of $E_n^p(.)$, namely K, L, M, N :

$$E_n^p(\lambda) = \begin{cases} K(\lambda) = \lambda^{n-2r} \sum\limits_{j=0}^{r} a_j \lambda^{2j} & p = [0, r] \\ L(\lambda) = \lambda^{1-n+2r} \sqrt{|\lambda^2 - h^2|} \sum\limits_{j=0}^{r-1} a_j \lambda^{2j} & p = [r+1, n] \\ M(\lambda) = \lambda^{1-n+2r} \sqrt{|\lambda^2 - k^2|} \sum\limits_{j=0}^{r-1} a_j \lambda^{2j} & p = [n+1, 2n-r] \\ N(\lambda) = \lambda^{n-2r} \sqrt{|\lambda^2 - h^2|} \sqrt{|\lambda^2 - k^2|} \sum\limits_{j=0}^{r-2} a_j \lambda^{2j} & p = [2n-r+1, 2n] \end{cases} \tag{2}$$

where $r = \left\lfloor \frac{n}{2} \right\rfloor$. There are $r+1$ polynomials that belong to the family K, $n-r$ polynomials that belong to the families L and M and r polynomials that belong to family N. The

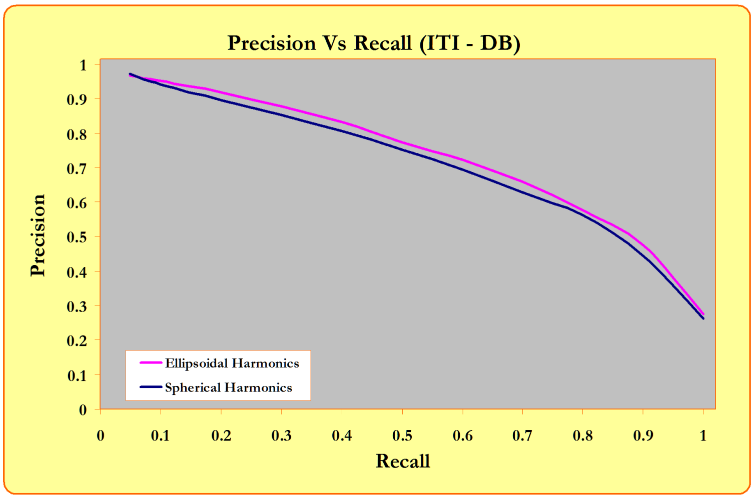

Figure 1: Precision-Recall Diagram in the ITI database.

parameters a_j for every ellipsoidal harmonics degree are easily computed using appropriate eigenanalysis. Detailed computational issues of Ellipsoidal Harmonics are above the scope of this paper and the reader is referred to [6, 7].

Suppose that $O_{all}(.)$ is a volumetric representation of a 3D object and ε is the best-fitted ellipsoid. The object is expressed in the Ellipsoidal Coordinate System and the R ellipsoids ε_i,$i = 1 \dots R$, which are homofocal to ε are defined. The intersection of every ellipsoid with the volumetric object produce the set of function $O_i(\lambda_2, \lambda_3)$, which are expanded into the ellipsoidal harmonics coefficients according to:

$$o^i_{np} = \int \int_S O_i(\lambda_2, \lambda_3) E^p_n(\lambda_2) E^p_n(\lambda_3) dS \tag{3}$$

where S is the surface of the ellipsoid ε_i. The values o^i_{np}, for $n = 0 \dots n_{max}$, $p = 0 \dots 2n$, $i = 1 \dots R$ forming the ellipsoidal harmonic descriptor vector.

Experimental Results

The proposed descriptor has been evaluated using the ITI 3D objects database [8], consisted of 544 objects classified in 12 categories and compared to the well-known approach of Gaussian with the Euclidean distance transform of the surface using spherical harmonics [4]. The evaluation performed using the precision-recall diagrams, where precision is defined as the ratio of the relevant retrieved objects and the total number of the retrieved objects, and recall is the ratio of the relevant retrieved objects and the total relevant objects in the database. Figure 1 presents the comparative performance between the two approaches. It is obvious that the Ellipsoidal Harmonic descriptor clearly outperforms the approach of [4].

Conclusions - Future Work

The evaluation of the Ellipsoidal Harmonic Descriptor seems promising. However, the approach requires extensions, that could increase its performance. The future work on this area involves the usage of more intrinsic functions $O_i(.)$ as inputs in the Ellipsoidal Harmonic analysis, and the combination of Ellipsoidal Harmonics with the rotation invariant features of Spherical Harmonics.

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